



Duration : 4 hr 30 min

7 points each problem

**Problem 1.**

Two circles  $\Gamma_1$  and  $\Gamma_2$  intersect at  $M$  and  $N$ .

Let  $l$  be the common tangent to  $\Gamma_1$  and  $\Gamma_2$  so that  $M$  is closer to  $l$  than  $N$  is. Let  $l$  touch  $\Gamma_1$  at  $A$  and  $\Gamma_2$  at  $B$ . Let the line through  $M$  parallel to  $l$  meet the circle  $\Gamma_1$  again at  $C$  and the circle  $\Gamma_2$  again at  $D$ .

Lines  $CA$  and  $DB$  meet at  $E$ ; lines  $AN$  and  $CD$  meet at  $P$ ; lines  $BN$  and  $CD$  meet at  $Q$ .

Show that  $EP = EQ$ .

**Problem 2.**

Let  $a, b, c$  be positive real numbers such that  $abc = 1$ . Prove that

$$\left(a - 1 + \frac{1}{b}\right) \left(b - 1 + \frac{1}{c}\right) \left(c - 1 + \frac{1}{a}\right) \leq 1.$$

**Problem 3.**

Let  $n \geq 2$  be a positive integer. Initially, there are  $n$  fleas on a horizontal line, not all at the same point.

For a positive real number  $\lambda$ , define a *move* as follows:

choose any two fleas, at points  $A$  and  $B$ , with  $A$  to the left of  $B$ ;

let the flea at  $A$  jump to the point  $C$  on the line to the right of  $B$  with  $BC/AB = \lambda$ .

Determine all values of  $\lambda$  such that, for any point  $M$  on the line and any initial positions of the  $n$  fleas, there is a finite sequence of moves that will take all the fleas to positions to the right of  $M$ .



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**Problem 4.**

A magician has one hundred cards numbered 1 to 100. He puts them into three boxes, a red one, a white one and a blue one, so that each box contains at least one card.

A member of the audience selects two of the three boxes, chooses one card from each and announces the sum of the numbers on the chosen cards. Given this sum, the magician identifies the box from which no card has been chosen.

How many ways are there to put all the cards into the boxes so that this trick always works? (Two ways are considered different if at least one card is put into a different box.)

**Problem 5.**

Determine whether or not there exists a positive integer  $n$  such that

$n$  is divisible by exactly 2000 different prime numbers, and

$2^n + 1$  is divisible by  $n$ .

**Problem 6.**

Let  $AH_1$ ,  $BH_2$ ,  $CH_3$  be the altitudes of an acute-angled triangle  $ABC$ . The incircle of the triangle  $ABC$  touches the sides  $BC$ ,  $CA$ ,  $AB$  at  $T_1$ ,  $T_2$ ,  $T_3$ , respectively. Let the lines  $l_1$ ,  $l_2$ ,  $l_3$  be the reflections of the lines  $H_2H_3$ ,  $H_3H_1$ ,  $H_1H_2$  in the lines  $T_2T_3$ ,  $T_3T_1$ ,  $T_1T_2$ , respectively.

Prove that  $l_1$ ,  $l_2$ ,  $l_3$  determine a triangle whose vertices lie on the incircle of the triangle  $ABC$ .